ONE-DIMENSIONAL UNSTEADY TURBULENT FLOW IN A PIPELINE WITH KARMAN RHEOLOGY TAKEN INTO ACCOUNT

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A dependence is found for the hydraulic flow drag on the Reynolds number and nonstationarity parameter.

Unsteady flow in a pipeline in a hydraulic formulation is described by one-dimensional equations for the mass flow rate and pressure as a function of the time and the longitudinal coordinate. An unknown friction stress on the internal pipe surface, for which the expression is taken exactly the same as in a stationary stream [1], enters into these equations. The friction stress here depends just on the mean stream velocity over the section and is independent of the nonstationarity parameter $K_n = d/W^2|\partial W/\partial t|$ and the other unsteady flow characteristics. Experiments show that such an assumption is valid for $K_n < 0.02$ [2]. For large values of the quantity K_n a noticeable discrepancy is noted between the computational and experimental data.

In this connection, it is proposed to use results of solving an axisymmetric problem on turbulent unsteady fluid motion in a constant radius pipe to calculate the friction stress in a one-dimensional model. A two-layer stream-viscous layer scheme near the inner pipe surface and a turbulent core in the rest of the stream is assumed for finding the velocity profile. A Newtonian rheological model according to which $\tau = \mu(\partial u/\partial y)$, $0 \le y \le \delta$ is considered in the viscous layer.

The rheological model of a fluid given by the Karman formula

$$\tau = -\rho\varkappa^2 \frac{\left|\frac{\partial v}{\partial r}\right|^3}{\left(\frac{\partial^2 v}{\partial r^2}\right)^2} \frac{\partial v}{\partial r}, \quad 0 \leqslant r \leqslant R-\delta \tag{1}$$

is introduced into the turbulent stream core. Such a rheological approach to describing the average turbulent stream in the stream core is developed in [3, 4]. If the Karman relationship (1) is taken as a rheological law for a turbulent medium in the stream core in a pipe, then it can be used to examine both stationary and nonstationary motion. Here (1) can be obtained on the basis of dimensional analysis [4, 5].

The fluid motion in a viscous layer is described by the following equation

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}, \quad u = u(y, t), \; t > 0, \quad 0 \leqslant y \leqslant \delta(y, t). \tag{2}$$

The equation of motion in the core of a turbulent stream has the form

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}(r\tau), \quad v = v(r, t), \quad t > 0, \quad 0 \leqslant r \leqslant R-\delta. \tag{3}$$

Here τ is the friction stress that is calculated by means of (1).

Let us formulate the boundary conditions to find the solutions of (2) and (3). The velocity on the inner pipe surface equals zero

$$u(0, t) = 0. \tag{4}$$

Three conditions must be given on the boundary of the viscous layer and the turbulent stream core. Two reflect continuity of the velocity and the friction stress. We have

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$$u|_{y=\delta-0}=v|_{y=\delta+0}, \qquad \mu\frac{\partial u}{\partial y}\Big|_{y=\delta-0}=-\rho\varkappa^2\frac{\left|\frac{\partial v}{\partial r}\right|^3}{\left(\frac{\partial^2 v}{\partial r^2}\right)^2}\frac{\partial v}{\partial r}\Big|_{\substack{y=\delta+0\\ r=R-\delta}}.$$

The equation of motion in the core is of third order, consequently, still another condition must be given to find the solution. We take as such a condition [6]

$$\frac{\partial u}{\partial y}\Big|_{y=\delta-0}=-K\frac{\partial v}{\partial r}\Big|_{\substack{y=\delta+0\\ r=R-\delta}}. \tag{5}$$

A finite discontinuity of the velocity derivatives is thereby assumed on the turbulent core boundary. On the pipe axis

$$v|_{r=0}=v_0. \tag{6}$$

From dimensionality considerations the viscous layer thickness is introduced according to the relationships

$$\delta=\frac{\alpha\nu}{|u_*|}, \quad u_*=\sqrt{\frac{|\tau_W|}{\rho}}\operatorname{sign}\tau_W. \tag{7}$$

Applying the Slezkin–Targ method that is based on the assumption that the local acceleration $\partial v/\partial t$ can be replaced by the mean acceleration $\partial W/\partial t$ over the section, we write the equation of motion in the stream core in the form

$$\frac{\partial p}{\partial x}+\rho\frac{\partial W}{\partial t}=-\frac{1}{r}\frac{\partial}{\partial r}(r\tau), \quad 0\leqslant r\leqslant R-\delta. \tag{8}$$

A linear tangential stress profile is obtained in the stream core from (8)

$$\frac{\tau_\ell}{\tau}=\frac{R-\delta}{r}, \quad \tau_\ell=\tau|_{r=R-\delta}.$$

Hence

$$\varkappa\frac{\left(\frac{\partial v}{\partial r}\right)^2}{\frac{\partial^2 v}{\partial r^2}}=\pm\sqrt{\frac{|\tau_\ell|r}{\rho(R-\delta)}}. \tag{9}$$

To the accuracy of terms on the order of δ/R the solution of (9) with the boundary conditions (5) and (6) has the form

$$v=v_0+\frac{\operatorname{sign}\tau_\ell}{\varkappa}\sqrt{\frac{|\tau_\ell|}{\rho}}\left[\ln\left(1-\sqrt{\frac{r}{R}}\right)-\sqrt{\frac{r}{R}}\right]. \tag{10}$$

To eliminate v_0 we take the average of the profile (10) by means of the formula

$$W=\frac{2}{R^2}\int_0^R vr\,dr.$$

We then obtain to the accuracy of terms of order δ/R

$$W=v_0+\frac{2c_1\operatorname{sign}\tau_\ell}{\varkappa}\sqrt{\frac{|\tau_\ell|}{\rho}}, \tag{11}$$

where

$$c_1=\int_0^1\left(\sqrt{\xi}+\ln|\sqrt{\xi}-1|\right)\xi d\xi=-0,642.$$

The value of the velocity on the stream core boundary [7] is found from (10) and (11)

$$v|_{r=R-\delta}=W+\frac{\operatorname{sign}\tau_\ell}{\varkappa}\sqrt{\frac{|\tau_\ell|}{\rho}}\left(1-2c_1+\ln\frac{K\nu}{2\varkappa R}\sqrt{\frac{\rho}{|\tau_\ell|}}\right). \tag{12}$$

To solve (2), a change of variables is performed in the boundary layer according to the formulas $\eta = y/\delta(t_1)$, $t_1 = t$, which permit representation of the acceleration in the form of a sum of two components

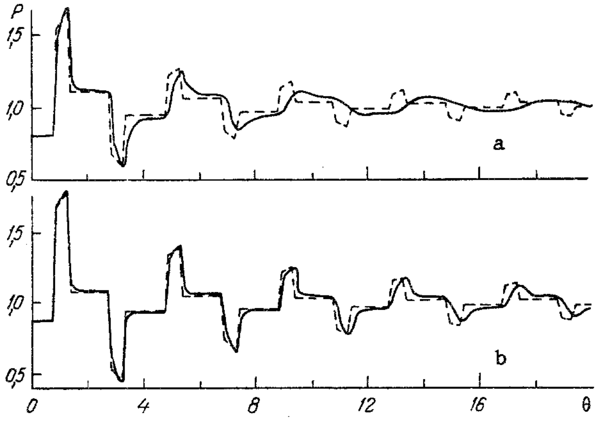

Fig. 1. Influence of the Reynolds number on the time dependence of the pressure during a hydraulic shock: X = 0.25; R_n = 200; θ_0 = 0.15; a) Re_1 = 10,000; b) Re_1 = 50,000.

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t_1} - \frac{\delta'}{\delta}\eta\frac{\partial u}{\partial \eta}. \tag{13}$$

The prime denotes differentiation with respect to the time. The first component in the right side of the relationship (13) is determined by the relative change in the velocity profile, and the second by the change in the boundary layer thickness.

Let us introduce the characteristic time of a nonstationary process T into the consideration (the time to close a slide gate, etc.). This same time will also be characteristic for the change in boundary layer thickness. The characteristic time to reconstruct the velocity profile in the boundary leyer is of the order δ^2/ν, consequently if $T \gg \delta^2/\nu$, then the first component in the relationship (13) can be neglected, then (2) is simplified and after the substitution $\varphi = \partial u/\partial \eta$ goes over into

$$\frac{\partial \varphi}{\partial \eta} + \frac{\delta\delta'}{\nu}\eta\varphi = \frac{\delta^2}{\mu}\frac{\partial p}{\partial x}, \quad 0 \leqslant \eta \leqslant 1. \tag{14}$$

On the pipe surface the function φ equals

$$\varphi|_{\eta=0} = \frac{\tau_W\delta}{\mu}. \tag{15}$$

The solution of (14) with the boundary condition (15) is sought in the form

$$\varphi = \sum_{n=0}^{\infty} a_n\eta^n. \tag{16}$$

The relationships

$$a_{2k} = \frac{(-1)^k}{2^k k!}\left(\frac{\delta\delta'}{\nu}\right)^k\frac{\tau_W\delta}{\mu}; \quad a_{2k+1} = \frac{(-1)^k}{(2k+1)!!}\left(\frac{\delta\delta'}{\nu}\right)^k\frac{\delta^2}{\mu}\frac{\partial p}{\partial x}$$

are obtained for values of the coefficients a_n. For $T \gg \delta^2/\nu$ the series (16) can be limited to several of the first terms

$$\frac{\partial u}{\partial \eta} = \varphi \approx \frac{\tau_W\delta}{\mu} + \frac{\delta^2}{\mu}\frac{\partial p}{\partial x}\eta - \frac{1}{2}\frac{\delta^2\delta'}{\nu^2}\frac{\tau_W}{\rho}\eta^2. \tag{17}$$

Integrating (17) with respect to η by using condition (4), we obtain for the value of the velocity on the boundary with the stream core

$$u|_{\eta=1} = \frac{\tau_W\delta}{\mu}\left(1 - \frac{1}{6}\frac{\delta\delta'}{\nu}\right) + \frac{1}{2}\frac{\delta^2}{\mu}\frac{\partial p}{\partial x}. \tag{18}$$

To eliminate the pressure gradient $\partial p/\partial x$ from (18), the equation in the stream core (8) is used. After taking the average of the components over the core section, we have

$$\frac{1}{\mu}\frac{\partial p}{\partial x} = -\frac{1}{\nu}\frac{\partial W}{\partial t} - \frac{2\tau_\ell}{\mu R}. \tag{19}$$

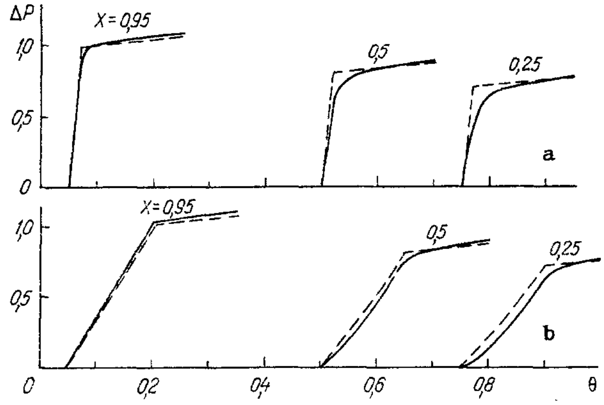

Fig. 2. Influence of the nonstationary level on pressure wave front damping: Re_1 = 10,000; R_n = 200; a) θ_0 = 0.015; b) θ_0 = 0.15.

Since $\tau = \mu\ \partial u/\partial y = \mu/\delta\ \partial u/\partial\eta$ in the boundary layer, then taking account of the continuity condition for τ during passage from the core into the boundary layer, we obtain from the solution (17)

$$\frac{\tau_\ell}{\mu} = \frac{1}{\delta}\varphi\Big|_{\eta=1} = \frac{\tau_W}{\mu}\left(1 - \frac{1}{2}\frac{\delta\delta'}{\nu}\right) + \frac{\delta}{\mu}\frac{\partial p}{\partial x}. \quad (20)$$

Solving the relationships (19) and (20) for τ_ℓ and $\partial p/\partial x$, we have to the accuracy of terms of the order δ/R

$$\frac{1}{\mu}\frac{\partial p}{\partial x} \approx -\frac{1}{\nu}\frac{\partial W}{\partial t} - \frac{2\tau_W}{\mu R}\left(1 - \frac{1}{2}\frac{\delta\delta'}{\nu}\right), \quad (21)$$

$$\tau_\ell \approx \tau_W\left(1 - \frac{\delta}{u_*^2}\frac{\partial W}{\partial t} - \frac{1}{2}\frac{\delta\delta'}{\nu}\right). \quad (22)$$

Substituting (21) into (18), we obtain to the accuracy mentioned

$$u|_{y=\delta}\ \frac{\tau_W\delta}{\mu}\left(1 - \frac{1}{6}\frac{\delta\delta'}{\nu}\right) - \frac{\delta^2}{2\nu}\frac{\partial W}{\partial t}. \quad (23)$$

The continuity condition for the velocity on the boundary of the core and the boundary layer permits equating the right sides of (12) and (23). Then taking account of (23), we write

$$\frac{\tau_W\delta}{\mu}\left(1 - \frac{Z}{3}\right) - \frac{\delta^2}{2\nu}\frac{\partial W}{\partial t} = W + \frac{u_*}{\varkappa}\sqrt{1 - \frac{\delta}{u_*^2}\frac{\partial W}{\partial t} - Z}\ \times$$
$$\times\left[1 - 2c_1 + \ln\frac{K\nu}{2\varkappa R|u_*|} - \frac{1}{2}\ln\left(1 - \frac{\delta}{u_*^2}\frac{\partial W}{\partial t} - Z\right)\right],$$
$$Z = \frac{1}{2}\frac{\delta\delta'}{\nu}. \quad (24)$$

Using (7), the parameter Z can be represented in the form

$$Z = -\frac{\alpha^2\nu}{2}\frac{u_*'}{u_*^2}. \quad (25)$$

Upon satisfaction of the condition $T \gg \delta^2/\nu$ the parameters Z and $\delta/u_*^2\ \partial W/\partial t$ can be less than one, and therefore, linearization of (24) is allowable. Then, by using (7) and (25), we obtain from (24)

$$\frac{\alpha^2\nu}{2u_*^3}\frac{\partial u_*}{\partial t}\left(\frac{\alpha}{3} + \frac{c_1}{\varkappa} - \frac{1}{2\varkappa}\ln\frac{\alpha\nu}{d|u_*|}\right) = \frac{W}{u_*} + \frac{1}{\varkappa} -$$
$$-\left(1 - \frac{\alpha\nu}{2u_*^3}\frac{\partial W}{\partial t}\right)\left(\alpha + \frac{2c_1}{\varkappa} - \frac{1}{\varkappa}\ln\frac{\alpha\nu}{d|u_*|}\right). \quad (26)$$

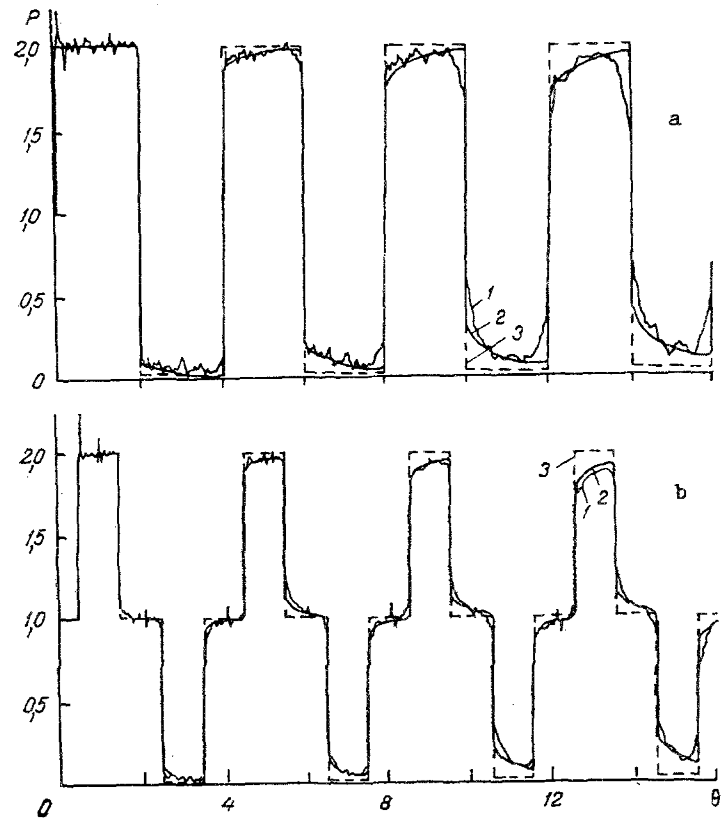

Fig. 3. Comparison of the results obtained with the Kholmbou and Rulo experiment [12] on a hydraulic shock for X = 1 (a) and X = 0.5 (b); (Re_1 = 6172; R_n = 1.017; θ_0 = 0.015): 1) experiment; 2) computation by the model proposed; 3) curves obtained on the basis of the quasistationary hypothesis.

The differential equation (26) permits determination of the dynamic velocity u_* as a function of the mean velocity and acceleration of the stream and can be used to close the system of one-dimensional equations [1] without involving the hypothesis of quasistationarity

$$\frac{\partial p}{\partial t}+\rho c^2\frac{\partial W}{\partial x}=0,\quad \frac{\partial p}{\partial x}+\rho\frac{\partial W}{\partial t}=-\frac{\rho u_*|u_*|}{2d}. \tag{27}$$

Equation (26) permits obtaining a relationship to find the hydraulic resistance coefficient for a nonstationary flow λ_{ns} which is introduced by the relationship

$$\left|\frac{W}{u_*}\right|=\sqrt{\frac{8}{\lambda_{ns}}}. \tag{28}$$

The desired equation is obtained in the form

$$A+B\ln \mathrm{Re}\sqrt{\lambda_{ns}}=\frac{1}{\sqrt{\lambda_{ns}}}+Z(A_1+B_1\ln \mathrm{Re}\sqrt{\lambda_{ns}})+\frac{K_a}{\mathrm{Re}\,\lambda_{ns}^{3/2}}(A_2+B_2\ln \mathrm{Re}\sqrt{\lambda_{ns}}). \tag{29}$$

After substituting u_* from (28) into (25), the parameter Z takes the form

$$Z=-\left(\frac{2\alpha}{\mathrm{Re}\sqrt{\lambda_{ns}}}\right)^2\frac{d^2}{\nu}\frac{\partial}{\partial t}(\ln \mathrm{Re}\sqrt{\lambda_{ns}}).$$

The parameters A, B, A_1, B_1, A_2, B_2 are expressed in terms of the two constants α and κ introduced earlier.

Equation (29) is an analogue of the Nikuradze relationship for the hydraulic drag coefficient with nonstationary "additions" proportional to the parameters Z and K_n. If we set $\alpha = K/\kappa$, then upon going over to the stationary mode the dependence (29) goes over into the Nikuradze relations, where its coefficients will have the same numerical values as in the stationary case. For the values $\kappa = 0.406$ and $K = 4.94$ taken in [6], we have $A = -0.768$; $B = 0.871$; $A_1 = -0.386$; $B_1 = 0.435$; $A_2 = 389$; $B_2 = 48.7$. The approximate expression for Z in terms of the nonstationarity parameter and the Reynolds number has the form

$$Z \approx \frac{4}{\lambda_{ns}} \left(\frac{K}{\varkappa} \right)^2 \frac{K_n}{\mathrm{Re}}.$$

Now both the last components in the relationship (29) are proportional to the factor K_n/Re. For sufficient smallness of this factor the expression (29) goes over into the Nikuradze relationship for the hydraulic drag coefficient for stationary flow, i.e., the hypothesis of quasistationarity is valid. Numerical estimates yield the condition for applicability of the quasistationarity hypothesis: $|K_n|/\mathrm{Re} < 3 \cdot 10^{-7}$. Therefore, the errors yielded by the quasistationarity hypothesis grow as the nonstationarity parameter increases and the Reynolds number diminishes. These deductions are confirmed by experimental data [8, 9] and the results of computations [10].

The mathematical model obtained, that consists of the system of equations (26) and (27), was used to compute transients in a pipeline. Constant pressure at one end of the pipe while a piecewise-linear change in the velocity due to slide gate motion was given at the other, were taken as boundary conditions. Stationary flow was considered as the initial conditions for the transient.

After passage to dimensionless quantities, (27), (26) and the relationship for u_* from (7) take the form

$$\frac{\partial P}{\partial \theta} + \frac{\partial U}{\partial X} = 0, \quad \frac{\partial P}{\partial X} + \frac{\partial U}{\partial \theta} = -D_n \mathrm{Re}_1 T_W,$$

$$\frac{a_2}{V^3} \frac{\partial V}{\partial \theta} (b_1 + \ln |V|) = a_1 \frac{U}{V} + 1 - \left(1 - \frac{a_3}{V^3} \frac{\partial U}{\partial \theta} \right) (b_2 + \ln |V|),$$

$$T_W = \frac{4}{\mathrm{Re}_1^2} V |V|.$$

The coefficients a_1, a_2, a_3, b_1, b_2 are calculated in terms of the Reynolds number Re_1 determined by means of the characteristic velocity, the dissipation parameter D_n, and the constants K and κ.

The initial and boundary conditions in dimensionless variables have the following form

$$P(X, 0) = P_0 - \frac{\lambda_c}{8} D_n \mathrm{Re}_1 U_0 |U_0| X, \quad U(X, 0) = U_0,$$

$$P(0, \theta) = P_0, \quad U(1, \theta) = \begin{cases} U_0 - \dfrac{U_0 - U}{\theta_0} \theta & \text{for } 0 \leqslant \theta \leqslant \theta_0, \\ U_K & \text{for } \theta \geqslant \theta_0. \end{cases}$$

The passage over to dimensionless variables exposes the governing parameters of the problem which turn out to be the Reynolds number Re_1, the damping parameter R_n ($R_n = D_n \mathrm{Re}_1$), and the dimensionless time of gate activation θ_0. The damping parameter is proportional to the ratio between the wave path time along the pipeline and the perturbation relaxation time in the turbulent core, consequently, it characterizes damping of the process. The time of gate activation θ_0 characterizes the nonstationarity level.

A series of computations of the hydraulic shock for values $\mathrm{Re}_1 = 20{,}000$, $R_n = 32$-800, $\theta_0 = 0.15$ of the governing parameters disclosed the dependence of the results of computations of the pressure fluctuations at fixed pipeline sections on the damping parameter R_n. The greater the damping parameter, the earlier does the divergence from the quasistationary model appear. The absolute value of this divergence is greater for small dampings.

"Blurring" of the pressure wave front, a phase shift and shorter duration of the transient as compared with the solution obtained by using the quasistationarity hypothesis (see Fig. 1) were observed in all the computations using the model being proposed.

The error given by the quasistationarity hypothesis grows as the Reynolds number diminishes, as was shown in examining the equation for the hydraulic drag coefficient (29).

This deduction was verified by a number of hydraulic shock computations (see figure) and the triggering of the filled pipeline. The Reynolds number varied within the band $Re_1 = 10^4$-$5\cdot 10^4$.

As is seen from Fig. 1, a computation by means of the model based on the hypothesis of quasistationarity (dashed curves) displayed a weak dependence of the solution on the Reynolds number. This is explained by the fact that the influence of the Reynolds number on the hydraulic drag coefficient is slight in the mentioned model. The solution obtained by means of the model proposed for the fixed parameters R_n and θ_0 depends substantially on the Reynolds number (solid curves).

As has been established above, the difference from the model based on the hypothesis of quasistationarity increases as the nonstationarity parameter K_n grows. It can be shown that the quasistationarity parameter K_n is inversely proportional to the dimensionless time of gate activation θ_0, i.e., the time of wave front formation, for a fixed damping parameter R_n.

The numerical solution of the hydraulic shock problem for different times θ_0 in the b and $\theta_0 = 0.015$-0.8 showed that the parameter θ_0 influences mainly the shape of the pressure wave at the beginning of the transient.

Application of the quasistationarity hypothesis results in the fact that the pressure growth time at any distance from the site of wave origination equals the front formation time θ_0. A computation by the proposed model showed that the pressure growth time increases with distance from the site of wave origination, where this increase will be the greater, the smaller the time θ_0. Shown in Fig. 2 is the pressure wave front at different pipeline sections (the slide gate is in the section X = 1). The solution obtained by the proposed model is displayed by solid lines and by the model bases on the quasistationarity hypothesis by dashed lines. As is seen from Fig. 2a, the pressure growth time obtained by the model proposed exceeds the quantity obtained on the basis of the quasistationarity hypothesis by several times at a sufficient distance from the slide gate.

This result is analogous to the result obtained when studying laminar unsteady fluid flow along a pipeline [11]. It is of practical importance since the pressure growth time is an essential parameter for the selection of the construction of regulating and protective apparatus.

To check out the model proposed, results were compared with the experimental data of Kholmbou and Rulo [12] on the hydraulic shock in a low-viscosity fluid (see Fig. 3). The initial flow mode was turbulent in these experiments.

Because of the small dimensions of the installation the damping in the experiments described was quite minute and the authors themselves considered the results to be described well by the Zhukovskii solution. Indeed, the solution based on application of the quasistationarity hypothesis differed slightly from the Zhukovskii solution and the experimental data in the first two periods are in good agreement with this model. However, a discrepancy is observed later which is especially noticeable in the section X = 0.5 (Fig. 3a). The model proposed yields good agreement with experiment.

Therefore, the one-dimensional model constructed for unsteady turbulent fluid flow in pipes permits taking account of the influence of the quasistationarity level on the friction drag. It is an expansion of the model based on the quasistationarity hypothesis.

NOTATION

x, r, longitudinal and radial coordinates; y, distance from the internal pipe surface; t, time; u, velocity along the pipe axis in the boundary layer; v, Reynolds average velocity along the pipe axis in the stream core; W, velocity averaged over the pipe section; p, pressure; τ, friction stress; u_*, dynamic velocity; μ, ν, dynamic and kinetic viscosities; R, d, pipe radius and diameter, respectively; δ, boundary layer thickness; ρ, density; c, speed of sound; λ_s, λ_{ns}, hydraulic drag coefficients for stationary and nonstationary flows; κ, K, α, constants. Dimensionless criteria: $Re = |W|d/\nu$, $Re_1 = |W_1|d/\nu$, Reynolds numbers defined by means of the running and characteristic mean velocities; K_n, nonstationary parameter; $D_n = \nu L/R^2c$, dissipation parameter; L, pipe length; R_n, damping parameter. Dimensionless quantities; $X = x/L$, longitudinal coordinate; $\theta = ct/L$, time; $U = W/W_1$, velocity; $P = p/(\rho c W_1)$, pressure; $V = Ru_*/\nu$, dynamic velocity; $T_W = \tau_W/(\rho W_1^2)$, tangential stress on the inner pipe surface; U_0, P_0, initial velocity and pressure; U_K, final velocity; and θ_0, time of gate activation.

LITERATURE CITED

1. I. A. Charnyi, Unsteady Motion of a Real Fluid in Pipes [in Russian], Moscow (1975).
2. V. M. Lyatkher, Turbulence in Hydraulic Constructions [in Russian], Moscow (1968).
3. V. V. Novozhilov, Theory of the Plane Turbulent Boundary Layer of an Incompressible Fluid [in Russian], Leningrad (1977).
4. M. V. Lur'e and N. A. Podoba, Dokl. Akad. Nauk SSSR, 279, No. 3, 570-575 (1984).
5. L. G. Loitsyanskii, Mechanics of Fluids and Gases [in Russian], Moscow (1970).
6. I. P. Ginzburg, Theory of Resistance and Heat Transfer [in Russian], Leningrad (1970).
7. I. G. Rozenberg, Trudy MINKh i GP, No. 186, Mechanics of Fluids and Gases [in Russian], 62-68, Moscow (1984).
8. S. B. Markov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 65-74 (1973).
9. S. V. Denisov, Inzh.-Fiz. Zh., 18, No. 1, 118-123 (1970).
10. B. S. Baibikov, G. A. Dreitser, V. G. Izosimov, and A. N. Shirshov, Trudy All-Union Polytechnic Correspondence Institute., Hydraulics [in Russian], 10, No. 3, 89-101 (1974).
11. G. D. Rozenberg and I. N. Buyanovskii, Unsteady Motion Equations of a Viscous Slightly Compressible Fluid in Pipes with the Influence of Nonstationarity on the Friction Force Taken Into Account. Appendix V to Unsteady Motion of a Real Fluid in Pipes by I. A. Charnyi [in Russian], 247-290, Moscow (1975).
12. E. L. Kholmbou and V. T. Rulo, Theoretical Principles of Engineering Computations [in Russian], No. 1 (1967), pp. 202-209.

STEADY-STATE TWO-DIMENSIONAL WAVES ON VERTICAL LIQUID FALLING FILMS AND THEIR STABILITY

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UDC 532.51

An investigation is reported of the stability of nonlinear conditions with respect to infinitely small perturbations; there is good agreement with experiment.

Their large interfacial contact areas and small thermal resistances make liquid films an effective means of carrying out interphase heat and mass transfer processes. It is well known that as a result of the instability of flows with flat free surfaces the nature of the motion of liquid films flowing down vertical walls is wavy even at small Reynolds numbers. The urgency to study these conditions arises as a result of the fact, in particular, that the presence of the waves has a considerable effect on the process of interfacial transfer through the free surface. Thus, in the desorption of slightly soluble gases the mass transfer coefficients may be increased by 100% or more as a result of the waves [1].

A special but important form of wavy flow consists of planar, steady-state periodic travelling waves. Their theoretical consideration is quite complicated, since it is necessary to solve a highly nonlinear boundary-value problem with a free boundary whose position is not known in advance. With the assumptions that the profiles of the longitudinal velocities are similar for any cross section x and any moment of time t:

$$u = 1{,}5\,\frac{q(x,t)}{h(x,t)}\left(2\,\frac{y}{h(x,t)} - \frac{y^2}{h^2(x,t)}\right)$$

and that the wavelengths are large, a system of equations has been derived in [2] for describing the behavior of perturbations on a film at moderate Reynolds numbers:

$$\frac{\partial q}{\partial t} + 1{,}2\,\frac{\partial}{\partial x}\left(\frac{q^2}{h}\right) = -\frac{3\nu q}{h^2} + gh + \frac{\sigma h}{\rho}\,\frac{\partial^3 h}{\partial x^3}, \quad \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0, \tag{1}$$

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